Jaynes-Cummings Model

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Jaynes-Cummings model

- It's a quantum optics model describing the interaction of a two-level atom with a single quantized mode of an optical cavity's electromagnetic field.
- Initially proposed by Edwin Jaynes and Fred Cummings in 1963 [1,2].
- First experimental demonstration in 1984 by Rempe, Walther, and Klein [3].
- It's been popular to study as it can be solved analytically and is easily extended. It also accurately predicts a wide range of experiments.
- Widely used in cavity QED and circuit QED, especially in relation to quantum information processing.

Outline

1. Derive the Jaynes-Cummings Hamiltonian

$$\hat{H}_{\mathrm{JC}} = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \hbar\lambda(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger)$$

2. Features of the model.
   - 2.1 Dressed states and the Jaynes-Cummings ladder.
   - 2.2 Vacuum-field Rabi oscillations.
   - 2.3 Collapse and revival of atomic oscillations.
3. Experimental observations of some of the model's features.

Quantizing the EM field

- We will derive the free field Hamiltonian $\hat{H}_{\text{field}} = \hbar\omega_c \hat{a}^\dagger \hat{a}$ by quantizing the electromagnetic field in a one-dimensional cavity. A more thorough 10-page derivation in three dimensions can be found in [4].
- We have to start somewhere. Let's start with Maxwell's equations in free space

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

which can be used to derive the homogeneous electromagnetic wave equation

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

Quantizing the EM field

- Now consider a one-dimensional cavity along the z-axis with perfectly conducting walls at $z = 0$ to $z = L$.
- We have to pick a polarization for the $\mathbf{E}$-field so might as well pick $\hat{\mathbf{x}}$ so that $\mathbf{E}(\mathbf{r}, t) = E_x(z, t)\hat{\mathbf{x}}$. The wave equation then reduces to

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial E_x}{\partial t} = 0$$

- This is easily solved by separation of variables with $E_x(z, t) = Z(z)T(t)$ yielding a solution

$$E_x(z, t) = \sqrt{\frac{2\omega_c^2}{V\epsilon_0}} q(t) \sin(kz)$$

Quantizing the EM field

- From Ampere's Law

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

we can find the magnetic field

$$B_y(z,t) = -\frac{1}{c^2} \int \frac{\partial E_x}{\partial t} dz = \sqrt{\frac{2\mu_0}{V}} \dot{q}(t) \cos(kz)$$

where V is the effective volume of the cavity, q is a time-dependent amplitude with units of length, and $k = m\pi/L$ for an integer $m > 0$.

Quantizing the EM field

- In this case, the classical field energy, which is equal to the Hamiltonian, is given by

$$
\begin{aligned}
H &= \frac{1}{2}\int dV \left(\epsilon_0 \mathbf{E}^2 + \frac{\mathbf{B}^2}{\mu_0}\right) \\
&= \frac{1}{2}\int dV \left(\epsilon_0 E_x^2(z,t) + \frac{B_y^2(z,t)}{\mu_0}\right) \\
&= \frac{1}{2}[\dot{q}^2(t) + \omega_c^2 q^2(t)]
\end{aligned}
$$

which looks like the Hamiltonian for a harmonic oscillator. (Surprise!)

Quantizing the EM field

- Now let's promote all our dynamical variables (q, $\dot{q}$, E_x, B_y, H) to operators, and we'll denote $\dot{q} \equiv p$, giving us

$$\hat{E}_x(z,t) = \sqrt{\frac{2\omega_c^2}{V\epsilon_0}}\hat{q}(t)sin(kz) \qquad \hat{B}_y(z,t) = \sqrt{\frac{2\mu_0}{V}}\hat{p}(t)\cos(kz)$$

$$\hat{H} = \frac{1}{2}[\hat{q}^2(t) + \omega_c^2\hat{q}^2(t)]$$

- Let's introduce creation and annihilation operators

$$\hat{a}(t) = \frac{1}{\sqrt{2\hbar\omega_c}}[\omega_c\hat{q}(t) + i\hat{p}(t)] \qquad \hat{a}^\dagger(t) = \frac{1}{\sqrt{2\hbar\omega_c}}[\omega_c\hat{q}(t) - i\hat{p}(t)]$$

Quantizing the EM field

- The electric and magnetic field can now be written as

$$\hat{E}_x(z,t) = E_0[\hat{a}(t) + \hat{a}^\dagger(t)]\sin(kz)$$

$$\hat{B}_y(z,t) = E_0[\hat{a}(t) - \hat{a}^\dagger(t)]\cos(kz)$$

- More importantly, we can write the Hamiltonian as

$$\hat{H} = \hat{H}_{\text{field}} = \hbar\omega_c\left[\hat{a}(t)\hat{a}^\dagger(t) + \frac{1}{2}\right] \approx \hbar\omega_c\hat{a}(t)\hat{a}^\dagger(t) = \hbar\omega_c\hat{a}\hat{a}^\dagger$$

- We can justify ignoring the zero-point energy due to redefining our zero of energy to be $\hbar\omega_c/2$ or if we assume that we have a lot of field quanta (recall that $\hat{n} = \hat{a}\hat{a}^\dagger$ is the number operator) such that $\hbar\omega_c/2$ is negligible.

Two-level atom Hamiltonian

- Let's denote our two the two levels of the atom by $|g\rangle$ for the ground state and $|e\rangle$ for the excited state, or in a vector representation

$$|g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad |e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- The Hamiltonian can then be written as

$$\begin{aligned}
\hat{H} &= E_g |g\rangle \langle g| + E_e |e\rangle \langle e| = \begin{pmatrix} E_e & 0 \\ 0 & E_g \end{pmatrix} \\
&= \frac{1}{2} \begin{pmatrix} E_g + E_e & 0 \\ 0 & E_g + E_e \end{pmatrix} + \frac{1}{2} \begin{pmatrix} E_e - E_g & 0 \\ 0 & -(E_e - E_g) \end{pmatrix} \\
&= \frac{1}{2}(E_g + E_e)\hat{\mathbb{1}} + \frac{1}{2}(E_e - E_g)\hat{\sigma}_z
\end{aligned}$$

Two-level atom Hamiltonian

- Writing the energy difference as $\hbar\omega_a = E_e - E_g$ where ω_a is the atomic transition frequency and shifting our zero of energy to $E_g + E_e$ because we only care about energy differences, we can write the atomic Hamiltonian as

$$\hat{H} = \hat{H}_{\text{atom}} = \frac{1}{2}\hbar\omega_a\hat{\sigma}_z$$

Interaction Hamiltonian

- As always, we'll start with $\hat{H} = -\hat{\mu} \cdot \hat{\mathbf{E}}$ so

$$\begin{aligned}\hat{H}_{\text{int}} &= -\hat{mu} \cdot E_0(\hat{a} + \hat{a}^\dagger)\sin(kz)\hat{\mathbf{x}} \\ &= \lambda\hat{\mu}(\hat{a} + \hat{a}^\dagger)\end{aligned}$$

where $\lambda = -\sqrt{\frac{\hbar\omega_c}{\epsilon_0 V}}\sin(kz)$.

- Recall the Pauli matrices

$$\hat{\sigma_1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{\sigma_2} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{\sigma_3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{\sigma}_+ = \hat{\sigma_1} + i\hat{\sigma_2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \hat{\sigma}_- = \hat{\sigma_1} - i\hat{\sigma_2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Interaction Hamiltonian

- We know that $\langle g|\hat{\mu}|g\rangle = 0$ and $\langle e|\hat{\mu}|e\rangle = 0$ due to parity. Then expanding $\hat{\mu}$ in terms of the basis states $\{|g\rangle, |e\rangle\}$, we get

$$\hat{\mu} = \mu |g\rangle \langle e| + \mu^\star |e\rangle \langle g| = \mu\hat{\sigma}_- + \mu\hat{\sigma}_+ = \mu(\sigma_- + \sigma_+)$$

  where we assumed without loss of generality that the matrix element $\mu = \mu_{ge} = \langle g|\hat{\mu}|e\rangle$ is real.

- Thus the interaction Hamiltonian can be written as

$$\hat{H}_{\text{int}} = \hbar\Omega(\hat{\sigma}_+ + \hat{\sigma}_-)(\hat{a} + \hat{a}^\dagger)$$

  where

$$\Omega = -\frac{\mu}{\hbar}\sqrt{\frac{\hbar\omega_c}{\epsilon_0 V}}\sin(kz)$$

Interaction Hamiltonian

- In the interaction picture, the operators evolve like

$$\hat{a}(t) = \hat{a}(0)e^{-i\omega_c t} \qquad \hat{a}^\dagger(t) = \hat{a}^\dagger(0)e^{i\omega_c t}$$

$$\hat{\sigma}_\pm = \hat{\sigma}_\pm(0)e^{\pm i\omega_a t}$$

- The Hamiltonian then becomes

$$\begin{aligned}\hat{H}_{\text{int}} &= \hbar\Omega(\hat{\sigma}_+\hat{a} + \hat{\sigma}_+\hat{a}^\dagger + \hat{\sigma}_-\hat{a} + \hat{\sigma}_-\hat{a}^\dagger) \\ &= \hbar\Omega(\hat{\sigma}_+\hat{a}e^{i(\omega_a-\omega_c)t} + \hat{\sigma}_+\hat{a}^\dagger e^{i(\omega_a+\omega_c)t} \\ &\qquad + \hat{\sigma}_-\hat{a}e^{-i(\omega_a+\omega_c)t} + \hat{\sigma}_-\hat{a}^\dagger e^{-i(\omega_a-\omega_c)t})\end{aligned}$$

- The $\hat{\sigma}_+\hat{a}^\dagger$ and $\hat{\sigma}_-\hat{a}$ terms vary much more rapidly than the other terms and so we invoke the *rotating wave approximation* and drop them. They are also unphysical.

Interaction Hamiltonian

- We are now left with the following interaction Hamiltonian

$$\hat{H}_{\text{int}} = \hbar\Omega(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger)$$

- The Jaynes-Cummings Hamiltonian is then

$$\hat{H}_{\text{JC}} = \hbar\omega_c\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\omega_a\hat{\sigma}_z + \hbar\Omega(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger)$$

just like we wanted.

Dressed states and the Jaynes-Cummings ladder

- The interaction Hamiltonian can only cause transitions of the type $|e\rangle|n\rangle \longleftrightarrow |g\rangle|n+1\rangle$ where these product states are referred to as the bare states of the Jaynes-Cummings model.
- For fixed n the dynamics of the system are confined to the two-dimensional space of product states $\{|e,n\rangle, |g,n+1\rangle\}$.
- In this basis, $\langle e,n|g,n+1\rangle = 0$, and the Hamiltonian can be written as

$$\hat{H}^{(n)} = \begin{pmatrix} n\hbar\omega_c + \frac{1}{2}\hbar\omega_a & \hbar\Omega\sqrt{n+1} \\ \hbar\Omega\sqrt{n+1} & (n+1)\omega_c - \frac{1}{2}\hbar\omega_a \end{pmatrix}$$

Dressed states and the Jaynes-Cummings ladder

- The energy eigenvalues of $\hat{H}^{(n)}$ are given by

$$E_{\pm}(n) = \left(n + \frac{1}{2}\right)\hbar\omega_c \pm \hbar\Omega_n(\Delta)$$

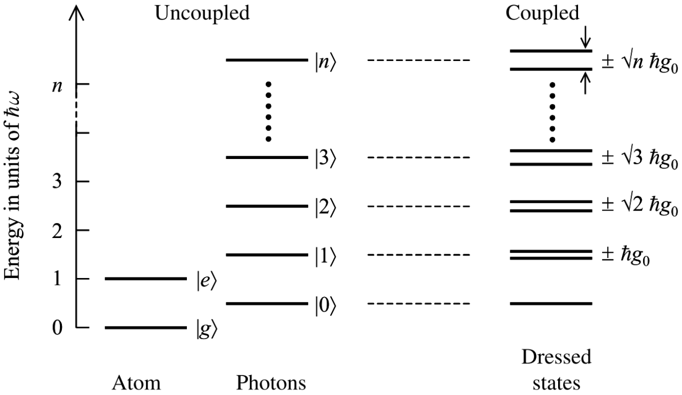
  where

$$\Omega_n(\Delta) = \sqrt{\Delta^2 + 4\Omega^2(n+1)}$$

  and $\Delta = \omega_a - \omega_c$ is the detuning.
- On resonance $\Delta = 0$ and $\Omega_n = 2\Omega\sqrt{n+1}$. So if we relabel $g_0 = 2\Omega$

$$E_{\pm} = \left(n + \frac{1}{2}\right)\hbar\omega_c \pm \sqrt{n+1}\hbar g_0$$

Dressed states and the Jaynes-Cummings ladder



The Jaynes-Cummings ladder. Note that the $\{|g, n\rangle, |e, n-1\rangle\}$ basis is used here so that $\Omega_n = \sqrt{n}\hbar g_0$.

Vacuum-field Rabi oscillations

- The JC Hamiltonian may be broken up into two commuting parts

$$\hat{H}_{\mathrm{JC}} = \hat{H}_{\mathrm{I}} + \hat{H}_{\mathrm{II}}$$

where

$$\hat{H}_{\mathrm{I}} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_a \hat{\sigma}_z$$

$$\hat{H}_{\mathrm{II}} = \hbar\Omega(\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger)$$

such that $[\hat{H}_{\mathrm{I}}, \hat{H}_{\mathrm{II}}] = 0$.
- All the dynamics are contained in the second part $\hat{H}_{\mathrm{II}}$.
- Let the initial state of the field-atom system be $|i\rangle = |e, n\rangle$ and the final state be $|f\rangle = |g, n+1\rangle$.
- The state vector may then be written

$$|\psi(t)\rangle = C_i\,|i\rangle + C_f\,|f\rangle$$

Vacuum-field Rabi oscillations

- The Schrodinger equation in the interaction picture states that

$$i\hbar \frac{d\left|\psi(t)\right\rangle}{dt} = \hat{H}_{\mathrm{II}}\left|\psi(t)\right\rangle$$

- This allows us to write down differential equations for the coefficients

$$\dot{C}_i = -i\Omega\sqrt{n+1}C_f$$
$$\dot{C}_f = -i\Omega\sqrt{n+1}C_i$$

or after plugging one into the other,

$$\ddot{C}_i + \Omega^2(n+1)C_i = 0$$

- We'll impose the initial conditions $C_i(0) = 1$ and $C_f(0) = 0$.

Vacuum-field Rabi oscillations

- Solving the pair of harmonic-oscillator-looking equations we get

$$C_i(t) = \cos\left(\Omega t\sqrt{n+1}\right)$$

$$C_f(t) = -i\sin\left(\Omega t\sqrt{n+1}\right)$$

- Thus the solution is

$$|\psi(t)\rangle = \cos\left(\Omega t\sqrt{n+1}\right)|e,n\rangle - i\sin\left(\Omega t\sqrt{n+1}\right)|g,n+1\rangle$$

- The probability the system remains in the ground state is

$$P_i(t) = |C_i(t)|^2 = \cos^2\left(\Omega t\sqrt{n+1}\right)$$

while the probability it makes a transition to the excited state is

$$P_i(t) = |C_i(t)|^2 = \sin^2\left(\Omega t\sqrt{n+1}\right)$$

Vacuum-field Rabi oscillations

- The atomic inversion is given by

$$W(t) = P_i(t) - P_f(t) = \cos\left(2\Omega t\sqrt{n+1}\right)$$

- These are Rabi oscillations with frequency $\omega(n) = 2\Omega\sqrt{n+1}$.
- We notice that even in the absence of light, i.e. $n = 0$, there is still a non-zero transition probability

$$W(t)|_{n=0} = \cos\left(2\Omega t\right)$$

- These vacuum-field Rabi oscillations are purely quantum mechanical and are the result of the atom spontaneously emitting a photon and absorbing it, then re-emitting it, etc.

Fock states

- To look at the collapse and revival of atomic oscillations, we will first have to look at Fock states and coherent states.
- Fock states, or $|n\rangle$, are eigenstates of the photon number operator

$$\hat{n}\,|n\rangle = n\,|n\rangle\,, \quad \sum_{n=0}^{\infty} |n\rangle\langle n| = 1, \quad \langle n|n'\rangle = \delta_{nn'}$$

- We know that

$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle \qquad \hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

and so excited states or Fock states can be written in terms of the vacuum state

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}\,|0\rangle$$

Coherent states

- Coherent states are eigenstates of the annihilation operator

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$$

- They have well-defined amplitudes $|\alpha|$ and phases $\operatorname{Arg}\alpha$. Since $\hat{a}$ is not Hermitian, the eigenvalues α may be complex and correspond to complex wave amplitudes in classical optics.
- We would like to express coherent states $|\alpha\rangle$ in terms of Fock states $|n\rangle$. To do so, we'll introduce the displacement operator

$$\hat{D}(\alpha) = e^{\alpha\hat{a}^\dagger - \alpha^\star\hat{a}}$$

- It is called so because it displaces the amplitude $\hat{a}$ by the complex number α

$$\hat{D}^\dagger(\alpha)\hat{a}\hat{D}(\alpha) = \hat{a} + \alpha$$

Coherent states

- This implies that

$$\hat{D}(-\alpha)\,|\alpha\rangle = |0\rangle$$

or that coherent states are simply displaced vacuum states

$$|\alpha\rangle = \hat{D}(\alpha)\,|0\rangle$$

- Recall the Baker-Campbell-Hausdorff formula

$$e^{\hat{A}+\hat{B}} = e^{-\frac{[\hat{A},\hat{B}]}{2}} e^{\hat{A}} e^{\hat{B}}$$

- We can now split the displacement operator like

$$\hat{D}(\alpha) = e^{-\frac{1}{2}|\alpha|^2} e^{\alpha \hat{a}^\dagger} e^{-\alpha^\star \hat{a}}$$

Coherent states

- Acting with it on the vacuum to displace it, we get that

$$|\alpha\rangle = \hat{D}(\alpha)\,|0\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\,|n\rangle$$

- The Fock representation shows that a coherent state has Poissonian photon statistics

$$P_n = |\langle n|\alpha\rangle|^2 = \frac{|\alpha|^{2n}}{n!} e^{-|\alpha|^2}$$

Collapse and revival of atomic oscillations

- Let's now consider more general (and interesting) dynamics.
- Let's assume the atom is initially in a superposition

$$|\psi(0)\rangle_{\text{atom}} = C_g |g\rangle + C_e |e\rangle$$

- And let's assume the field is initially in a coherent state

$$|\psi(0)\rangle_{\text{field}} = \sum_{n=0}^{\infty} C_n |n\rangle \, , \quad C_n = e^{-\frac{1}{2}|\alpha|^2} \frac{\alpha^n}{\sqrt{n!}}$$

- The initial atom-field state is then

$$|\psi(0)\rangle = |\psi(0)\rangle_{\text{atom}} \otimes |\psi(0)\rangle_{\text{field}}$$

Collapse and revival of atomic oscillations

- The solution to Schrodinger's equation is now

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} \left\{ \left[C_e C_n \cos\left(\Omega t\sqrt{n+1}\right) - iC_g C_{n+1} \sin\left(\Omega t\sqrt{n+1}\right)\right] |e\rangle \right.$$
$$\left. + \left[-iC_e C_{n-1} \sin\left(\Omega t\sqrt{n}\right) + C_g C_n \cos\left(\Omega t\sqrt{n}\right)\right] |g\rangle \right\} \otimes |n\rangle$$

- If we again take the case of $C_e = 1$, $C_g = 0$ then the solution may be written as

$$|\psi(t)\rangle = |\psi_g(t)\rangle\,|g\rangle + |\psi_e(t)\rangle\,|e\rangle$$

where

$$|\psi_g(t)\rangle\,|g\rangle = -i\sum_{n=0}^{\infty} C_n \sin\left(\Omega t\sqrt{n+1}\right)|n+1\rangle$$

$$|\psi_e(t)\rangle\,|e\rangle = \sum_{n=0}^{\infty} \cos\left(\Omega t\sqrt{n+1}\right)|n\rangle$$

Collapse and revival of atomic oscillations

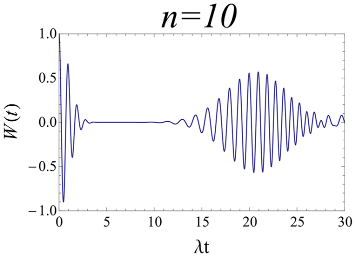
- The atomic inversion is now given by

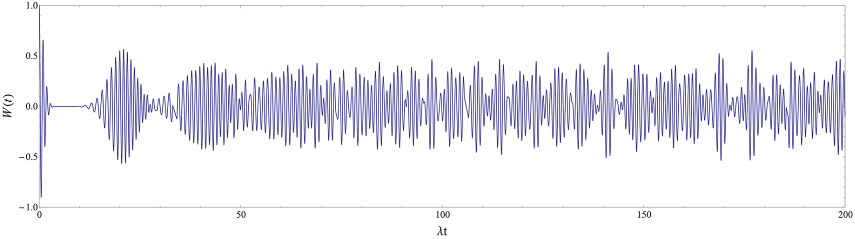
$$
\begin{aligned}
W(t) &= \langle \psi_e(t)|\psi_e(t)\rangle - \langle \psi_g(t)|\psi_g(t)\rangle \\
&= \sum_{n=0}^{\infty} |C_n|^2 \cos\left(2\Omega t\sqrt{n+1}\right) \\
&= \sum_{n=0}^{\infty} e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} \cos\left(2\Omega t\sqrt{n+1}\right)
\end{aligned}
$$

- The average photon number is $\bar{n} = |\alpha|^2$ and so we can write the inversion as

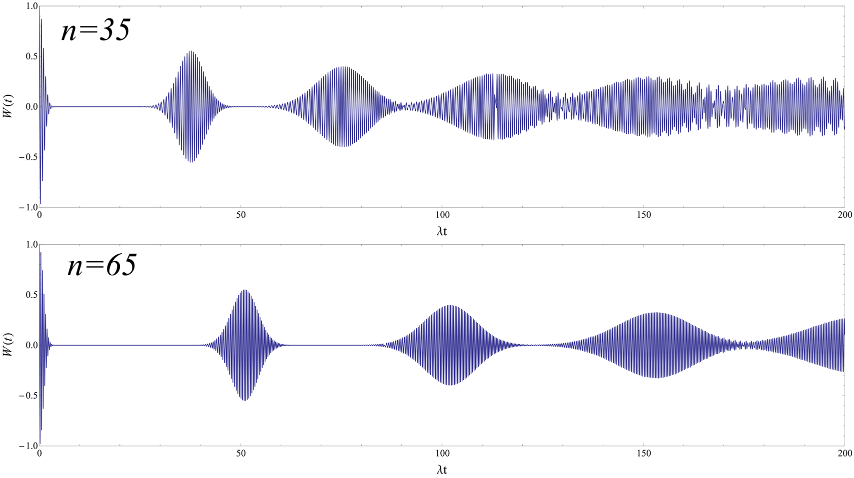
$$
W(t) = e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \cos\left(2\Omega t\sqrt{n+1}\right)
$$

Collapse and revival of atomic oscillations





Collapse and revival of atomic oscillations



First experimental observation of collapse and revival

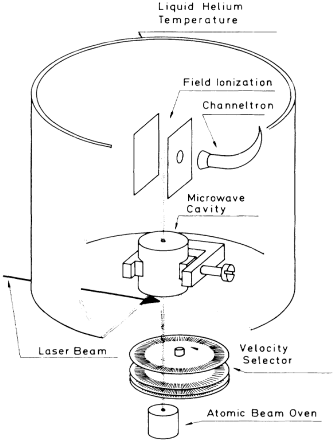


FIG. 1. Scheme of the experimental setup.

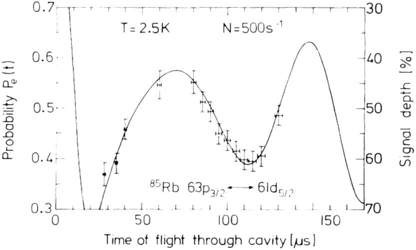


FIG. 3. The probability $P_e(t)$ of finding the atom in the upper maser level $63p_{3/2}$ for the cavity tuned to the $63p_{3/2} \leftrightarrow 61d_{5/2}$ transition of ^{85}Rb. The flux of Rydberg atoms is $N = 500\ \mathrm{s}^{-1}$.

[3] *Phys. Rev. Lett.* **58**(4), 353 (1987)

Coherent control of vacuum Rabi oscillations

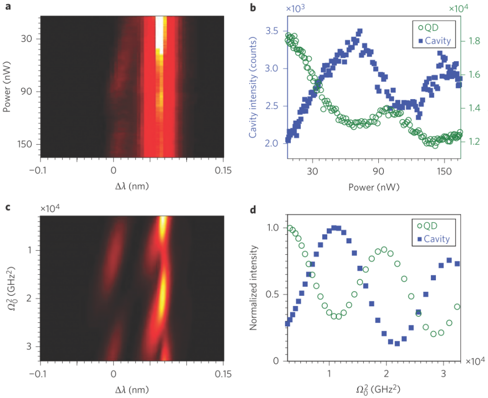


Figure 4 | Rabi oscillations. a, Measured reflection spectrum as a function of Stark laser power. **b**, Emission intensity at cavity resonance (blue squares) and at quantum-dot resonance (green circles), determined from the data in **a**. **c**, Calculated spectrum as a function of Stark power. The Stark field is expressed as a classical Rabi frequency with peak amplitude Ω_0. **d**, Calculated emission intensity at cavity resonance (blue squares) and quantum dot (QD) resonance (green circles). Intensities are normalized by their maximum value.

[5] *Nature Photon.* **8**, 858 (2014)

Observation of $\sqrt{n}$ nonlinearity in a cavity QED system

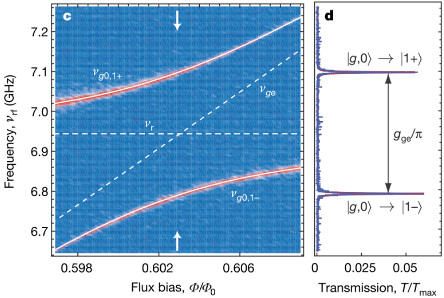


Figure 3 | Vacuum Rabi mode splitting with a single photon. a, Measured resonator transmission spectra versus normalized external flux bias, Φ/Φ_0 (bottom axis) and corresponding bias current I applied to a superconducting coil (top axis). Transmission T is colour coded: blue, low; red, high. The solid white line shows dressed state energies as obtained numerically, and the dashed lines indicate the bare resonator frequency ν_r as well as the qubit transition frequency ν_{ge}. **b**, Normalized resonator transmission T/T_{max} at $\Phi/\Phi_0 = 1/2$, as indicated with arrows in **a**, with a lorentzian line fit in red. **c**, Resonator transmission T versus Φ/Φ_0 close to degeneracy. **d**, Vacuum Rabi mode splitting at degeneracy, with lorentzian line fit in red.

[6] *Nature* **454**, 315 (2008)

Observation of $\sqrt{n}$ nonlinearity in a cavity QED system

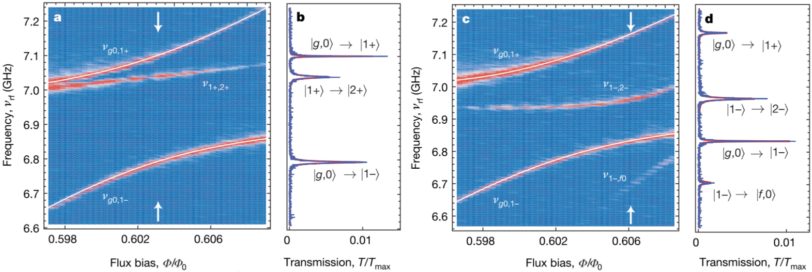


Figure 4 | Vacuum Rabi mode splitting with two photons. a, Cavity transmission T as in Fig. 3 with an additional pump tone applied to the resonator input at frequency $\nu_{g0,1+}$ populating the $|1+\rangle$ state. **b**, Spectrum at $\Delta = 0$, indicated by arrows in **a**. **c**, Transmission T with a pump tone applied at $\nu_{g0,1-}$ populating the $|1-\rangle$ state. **d**, Spectrum at $\Phi/\Phi_0 \approx 0.606$, indicated by arrows in **c**. See text for details of pump tone nomenclature.

[6] *Nature* **454**, 315 (2008)

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