The quantum coin toss—observation of quantum superposition in a mesoscopic coin

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Abstract

Ever since Schrödinger proposed his famous thought experiment in 1935 in which a cat is put in a quantum superposition of alive and dead states [1], physicists and philosophers have argued about the possibility of macroscopic systems being placed into a quantum superposition of physically distinct states. While this has been done with many microscopic systems, experimental demonstrations for macroscopic systems remain elusive. We propose a scheme for cooling a large levitating dielectric object trapped in a high-finesse cavity by optical tweezers to its motional ground state then preparing it in a quantum superposition state. A subsequent measurement of the quantum state constitutes the world's most sophisticated coin toss.

1 Introduction

It is well known that nature's smallest constituents, elementary particles such as the electron and photon, all obey the laws of quantum mechanics. All physical systems are composed of elementary particles and so should subsequently obey the same laws—quantum mechanics in principle applies to arbitrarily large systems. Atoms are composite particles and have long been known to behave quantum mechanically however the reason we do not observe quantum phenomena at everyday scales is due to the Correspondence principle which states that physical systems behave classically in the limit of large quantum numbers [2]. But could a macroscopic object behave in a quantum manner under the right conditions?

Many composite systems have been shown to behave quantum mechanically. Buckyballs (C_60) have been shown to exhibit wave-particle duality [3] and larger fullerenes (up to 420 atoms) have been observed to quantum interference [4]. The diffraction of molecules through a slit has even been imaged in real time [5]. Additionally, even quasi-particle systems such as single surface plasmon polaritons have also been shown to exhibit wave-particle duality [6]. However, these systems are of course not macroscopic in any way.

Quantum phenomena in truly non-microscopic systems has been observed in superconducting quantum interference devices, which were successfully placed into a superposition of two states of quantized magnetic flux [7], and more recently in optomechanical systems. Mesoscale optomechanical resonators have been laser cooled to their quantum motional ground state defined by an average phonon occupancy number less than 1 with possible applications in the optical control of such resonators [8]. Going further, quantum zero-point fluctuations have been measured in such resonators near their ground state using resolved sideband laser cooling [9]. A truly macroscopic system, an acoustical resonator the width of a thin human hair, has also been successfully cooled to its ground state and single phonon were controllably excited [10]. There is no questions that these are highly impressive feats and do confirm that quantum mechanics is valid at length scales far beyond the quantum realm, however they do not necessarily answer Schrödinger's original though experiment.

As only proposals exist so far for probing macroscopic quantum superposition [11], we shall study a previously proposed scheme of preparing quantum superpositions of living organisms [12] applied to a miniature-sized dielectric Canadian toonie and conduct the world's most sophisticated coin toss.

2 Experimental proposal

To carry out the coin toss, we would fabricate a miniature-sized dielectric Canadian toonie although a smoothed out replica without the polar bear would probably make for an easier experiment. The coin is then optically levitated and harmonically trapped in a high-finesse optical cavity. The coin must be sub-wavelength sized due to light scattering decreasing the finesse of the cavity.

2.1 Cavity optomechanics

The harmonicity of the trap decouples the coin's center-of-mass (COM) coordinate from any relative degree of freedom and thus the coin's COM coordinate may be quantized as $\hat{z} = z_m(\hat{b}^\dagger + \hat{b})$ where $z_m = \sqrt{\hbar/2M\omega_t}$ and ω_t is the trap's frequency. $\hat{b}$ and $\hat{b}^\dagger$ are the annihilation and creation operators, respectively, for phonons. This position dependence is identical in form to the quantization of the $\langle x \rangle$ operator for the quantum harmonic oscillator and gives rise to the usual optomechanical coupling Hamiltonian $\hat{H}_{\text{OM}} = \hbar g(\hat{b}^\dagger + \hat{b})(\hat{a}^\dagger + \hat{a})$ where $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators for a resonant photon in the cavity and $g \sim \sqrt{n}$ where n is the number of photons in the cavity.

The system also consists of a dissipative portion parameterized by a cavity decay rate κ and mechanical damping rate γ. As the coin is optically levitated, thermal transfer does not contribute significantly to γ. For this experiment to work we require a good cavity ($\omega_t > \kappa$) and strong coupling ($g \gg \kappa, \gamma$) as the preparation of the quantum superposition state relies on the single-photon Fock states which may decay too quickly under weak coupling.

2.2 Laser cooling and trapping of the coin

A coin will translation and rotational motion which must both be cooled. To do this, we employ Laguerre-Gauss beams following a proposal by Bhattacharya and Meystre [13]. Laguerre-Gauss beams have rotational symmetry along their propagation axis and carry an intrinsic orbital angular momentum of $i\hbar$ per photon (independent of the angular momentum due to the polarization of light) [14]. This allows for the exchange of orbital angular momentum between a radiation field, thus a dielectric object placed along the propagation axis will experience a torque. Using two counter-propagating Lageurre-Gauss beams, both the coin's translational and rotational motion may be trapped and cooled. We note that this experiment would be easier to do with a nanosphere which may be trapped and cooled by optical tweezers.

2.3 Preparation of the quantum superposition state

Gaussian states may be somewhat easily prepared however preparing superposition states is more challenging. We shall denote the heads state $|H\rangle = |0\rangle$ and the tails state $|T\rangle = |1\rangle$ where $|0\rangle$ and $|1\rangle$ are the ground and excited state of the quantum harmonic oscillator with quantized $\hat{z}$ coordinate. We then wish to prepare the coin in some quantum superposition state $|\psi\rangle = (|H\rangle + |T\rangle)/\sqrt{2}$. A measurement of $|H\rangle$ would indicate that the coin is at the center of the cavity of a measurement of $|T\rangle$ would place the coin slightly off the center of the cavity.

Preparation of the quantum superposition state begins by frequency doubling a laser pulse and sending it through a nonlinear crystal with a high second-order susceptibility to initiate spontaneous parametric down conversion, a process of producing entangled photons. Then shine the cavity with this single-photon Fock state $|n = 1\rangle$. Part of this field will be reflected (which we will measure later) and part transmitted. You want to adjust the reflectance and transmittance such that half of the one-photon pulse makes it into the cavity.

Solving the quantum Langevin equations for the system (which we skip due to space restrictions) show that in the presence of a red-detuned laser, the coupling Hamiltonian swaps the state of light inside the cavity with the mechanical motional state, yielding the entangled state

$$\frac{1}{\sqrt{2}}\left(|\tilde{0}\rangle\,|H\rangle + |\tilde{1}\rangle\,|T\rangle\right)$$

where $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$ are displaced vacuum and one-photon light states in the output mode of the cavity.

A balanced homodyne measurement is then made of the reflected one-photon pulse and the driving field is switched off to preserve the superposition state. A homodyne measurement involves mixing the weak cavity signal with a stronger oscillator signal of the same frequency then filtering out both signals to observe the mixing product whose frequency is the sum or difference of the signal and oscillator. This homodyne measurement causes the motional state to collapse into $|\psi\rangle = c_0 |0\rangle + c_1 |1\rangle$ where c_0 and c_1 may be calculated numerically and so experimental conditions that satisfy $c_0 = c_1$ should be picked.

2.4 Measurement of the quantum superposition state

Making a measurement of this quantum superposition state will rely on measuring the coin's expected position $\langle \hat{z} \rangle$ which will oscillate at a frequency ω_t with amplitude proportional to z_m. If the system is in in a Gaussian state no oscillation should be observed and thus $\langle \hat{z} \rangle$ oscillation provide an unambiguous sign that a quantum superposition state had been successfully prepared. Driving the cavity with a blue-detuned laser tuned to the upper motional sideband will amplify this signal and allow for easier detection.

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